

Nitrogen pumpout calc

We ask how much nitrogen is present, and how long to pump out. Assume a light tube of PMMA that is saturated with N2 at 1 atm to start. We then pull a good vacuum on it at t=0, and compute the diffusion out through the surfaces

Solubility: again from NIST reference:

Experimental Data		
Henry's law solubility constants S for gases in PMMA		
Gas	Pressure/atm	$S/(\text{cm}^3(\text{STP})/\text{g atm})$
He	3.8–7.2	0.066
Ne	2.7	0.126
Ar	3.2–7.4	0.105 ± 0.063
Kr	2.6–4.6	0.122
N ₂	5.7–15.0	0.045 ± 0.028
CO ₂	4.0–19.0	0.260 ± 0.024

$$S_{\text{N2_PMMA}} := .045 \frac{\text{scc}}{\text{gm} \cdot \text{atm}} \quad M_{\text{a_N2}} := 28 \frac{\text{gm}}{\text{mol}} \quad \text{scc} = 4.464 \times 10^{-5} \text{ mol}$$

Light tube dims:

$$t_{\text{lt}} := 3 \text{ mm} \quad l_{\text{lt}} := 1.3 \text{ m} \quad r_{\text{lt}} := 53 \text{ cm} \quad \text{buffer support and SiPM plane are not included here}$$

Light tube area, volume, mass:

$$\begin{aligned} A_{\text{lt}} &:= 4\pi r_{\text{lt}} \cdot l_{\text{lt}} & V_{\text{lt}} &:= 2\pi r_{\text{lt}} \cdot l_{\text{lt}} \cdot t_{\text{lt}} & M_{\text{lt}} &:= V_{\text{lt}} \cdot \rho_{\text{PMMA}} \\ A_{\text{lt}} &= 8.658 \text{ m}^2 & V_{\text{lt}} &= 0.013 \text{ m}^3 & M_{\text{lt}} &= 15.585 \text{ kg} \end{aligned}$$

Mass, number of moles N2

$$\begin{aligned} M_{\text{N2_lt}} &:= S_{\text{N2_PMMA}} \cdot M_{\text{lt}} \cdot 1 \text{ atm} \cdot M_{\text{a_N2}} & M_{\text{N2_lt}} &= 0.877 \text{ gm} & & \text{(note purifier has 30 gm removal capacity, below)} \\ N_{\text{N2_lt}} &:= S_{\text{N2_PMMA}} \cdot M_{\text{lt}} \cdot 1 \text{ atm} & N_{\text{N2_lt}} &= 0.031 \text{ mol} & S_{\text{m}} &:= \frac{M_{\text{N2_lt}}}{M_{\text{lt}}} \quad S_{\text{m}} = 5.625 \times 10^{-5} \end{aligned}$$

Molar concentration, initial

$$C_{\text{i_N2}} := S_{\text{N2_PMMA}} \cdot \rho_{\text{PMMA}} \cdot 1 \text{ atm} \quad C_{\text{i_N2}} = 2.411 \frac{\text{mol}}{\text{m}^3}$$

Diffusion constant for N2 in polymers

$$D_{\text{N2_PMMA}} := 5 \cdot 10^{-9} \frac{\text{m}^2}{\text{s}} \quad \text{from: -->}$$

Solubilities and diffusion coefficients of carbon dioxide and nitrogen in polypropylene, high-density polyethylene, and polystyrene under high pressures and temperatures
H.Masuoka, et al.)

Finite thickness plane (1D) Fick's 2nd law solution:

diffusion constant:

$$D := D_{\text{N2_PMMA}}$$

for plate of half thickness h , and distance from plate centerplane x , and some time after initial vacuum application

$$h := 0.5 t_{\text{lt}} \quad x := h \quad h = 1.5 \text{ mm} \quad x = 1.5 \text{ mm}$$

time unit

$$u := 1 \text{ min}$$

range variable

$$t := 1u, 2u, \dots, 100u$$

For an infinite sheet of width $2l$, with initial concentration C_i suddenly exposed to a surface concentration of C_p , total (integrated) mass flow through surface at time t divided by the total for $t = \text{infinity}$:

$$l := h \quad l = 1.5 \text{ mm}$$

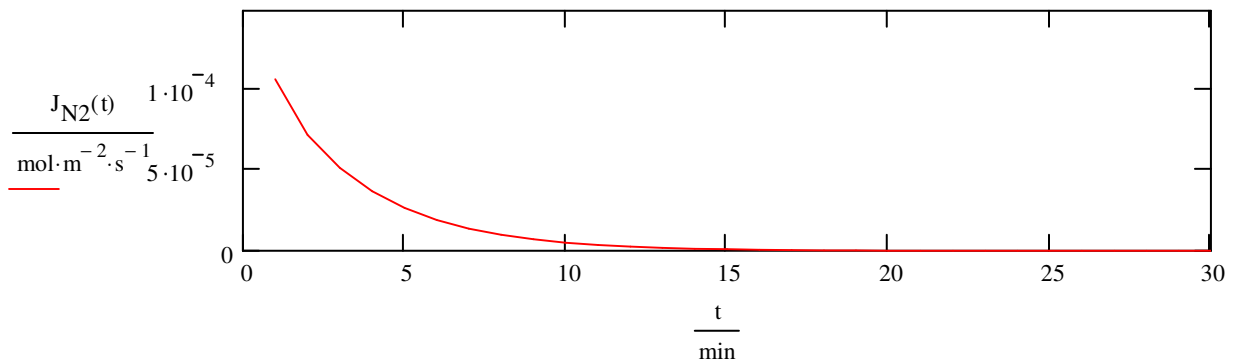
$$f_M(t) := 1 - \sum_{n=0}^{\infty} \left[\frac{8}{(2n+1)^2 \pi^2} e^{\left[\frac{-D \cdot (2n+1)^2 \pi^2 t}{4l^2} \right]} \right] \quad \text{Mathematics of Diffusion, Crank, eq. 4.18}$$

differentiating this with respect time gives us (dimensionless) flux (flow rate):

$$df_M(t) := \sum_{n=0}^{100} \left[2 \frac{D}{l^2} \cdot e^{\left[\frac{-D \cdot (2n+1)^2 \pi^2 t}{4l^2} \right]} \right] \quad \text{first few terms of summation are sufficient for long times}$$

and outgassing flux is then:

$$J_{N_2}(t) := V_{lt} \cdot C_{i_{N_2}} \cdot df_M(t)$$



$$\text{check --> } \int_{0s}^{1\text{day}} J_{N_2}(t) dt = 0.031 \text{ mol} \quad \text{initial gas load --> } N_{N_2_{lt}} = 0.031 \text{ mol}$$

95% removal time

$$t_{95} := 8\text{min} \quad \int_{0s}^{t_{95}} J_{N_2}(t) dt = 0.029 \text{ mol}$$

outgas rate after 0.1, 1 hr

$$t_{1hr} := \left(\frac{0.1}{1} \right) \text{hr}$$

$$J_{N_2}(t_{1hr}) = \left(\frac{1.933 \times 10^{-5}}{3.723 \times 10^{-13}} \right) \frac{\text{mol}}{\text{s}}$$

$$P_{1hr} := J_{N_2}(t_{1hr}) \cdot R \cdot T \quad P_{1hr} = \left(\frac{0.353}{6.802 \times 10^{-9}} \right) \frac{\text{torr} \cdot \text{L}}{\text{s}}$$